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# Adaptive Leader-Follower Formation Control for Autonomous Mobile Robots

Jing Guo<sup>1</sup>   Zhiyun Lin<sup>1</sup>   Ming Cao<sup>2</sup>   Gangfeng Yan<sup>1</sup>

**Abstract**— In this paper, adaptive formation control is addressed for a network of autonomous mobile robots in which there are only two leaders knowing the prescribed reference velocity while the others just play the role of followers. Assuming that each follower has only two neighbors to form a cascade interconnection, an adaptive formation control law is designed that allows each follower to achieve a specific triangular formation with its two neighbors without the need to know the velocity of its neighbors. With this scalable design approach, any expected geometric pattern of a group of  $n$  robots with two leaders can be realized by assigning an appropriate neighbor relationship and specifying a desired formation for each follower to reach. Both rigorous analysis and simulations are provided to demonstrate the effectiveness of the adaptive formation controller.

**Keywords:** Formation control, leader-following, autonomous robots

## I. INTRODUCTION

Formation control of a group moving agents has many applications such as transportation, surveillance, and search operations. Thus, it has attracted considerable attention in recent years. Graph rigidity as a tool to study vehicle formations was introduced in [1] and then has been widely explored within different contexts [2]–[7]. In addition to the work considering bidirectional interactions for rigid formations, there were also lots of works on unidirectional interactions in attaining rigid formations. For example, Cao et al. [10] analyzed the global convergence properties of three robots to a static formation with acyclic sensing graphs, while Anderson et al [11] addressed three vehicle formations with cyclic sensing graphs. Replacing single integrator models by double integrator models, Chen et al. [12] further investigated the three-coleader formation control problem via the backstepping method. On the other hand, in addition to the works focusing on distance-constrained formation descriptions and relative-position based formation control, Mariottini et al. [8] studied the leader-follower formation control problem with only bearing angles available, and Das

et al [9] considered formation control design using range only measurements. The problem of moving in formation with a prescribed velocity was also studied as extensions of static formations. One example is [13] where an adaptive control law was proposed based on passivity theory.

The leader-follower approach is one of the important methods for formation control problems [14]–[16]. The paper also studies the formation control problem in the leader-follower framework, but focuses on adaptive formation control without the need of knowing the leaders' velocity. For a group of autonomous mobile agents, we assume that there are two leaders knowing the reference velocity which governs the whole group's motion, but the others do not have access to the reference velocity information. The control objective is to devise a control law which enables the robots to form a rigid formation and to maintain the formation while moving as a whole with the desired reference velocity. We label the robots from 1 to  $n$  such that robots 1 and 2 are the leader robots and robots  $i$  ( $i = 3, \dots, n$ ) are the follower robots. The leader robots 1 and 2 can sense only the relative position of each other. The follower robot  $i$  has two neighbor robots  $i - 1$  and  $i - 2$ . Each follower robot measures its neighbors' positions in its own coordinate system. In the paper, we first introduce a control law for the leader robots to make them converge to a configuration with the prescribed distance and move with the reference velocity. Second, an adaptive scheme is proposed for each follower robot utilizing the measured relative position information so that it forms a desired triangular formation with its two leading neighbors. Thus, a group formation is realized by proper cascading triangular formations. However, for any follower and its two neighbors, the proposed control law also introduces another undesirable equilibrium formation, which is the collinear configuration. By investigating the stability properties of these equilibrium formations, it is shown that the triangular formation is stable while the collinear one is not. The obtained results extend the work [10] using triangularized structures in achieving static formations to the problem of controlling formations in motion. The benefit of using an adaptive scheme is clear. It reduces the cost for broadcasting the velocity information and assures that the robots can be adaptively recovered to the desired formation in the presence of an abrupt change of the reference velocity. On the other hand, the approach presented in the paper is scalable. With this scalable design approach, any expected geometric pattern of a group of  $n$  robots with two leaders can be realized by assigning an appropriate neighbor relationship and specifying a desired formation for each follower to reach.

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## II. PROBLEM STATEMENT

Consider a group of  $n$  identical mobile robots with dynamics

$$\dot{x}_i = u_i, \quad i = 1, \dots, n, \quad (1)$$

where  $x_i \in \mathbb{R}^2$  is the position of robot  $i$  in the plane and  $u_i \in \mathbb{R}^2$  is its velocity control.

We assume that each robot  $i$  carries an onboard sensor and is able to sense the relative position of robot  $j$  (namely,  $x_j - x_i$ ) when robot  $j$  is its neighbor. In our setup, the group has two leader robots with the knowledge of the reference velocity  $v_0$ , which is constant or piecewise constant. Two leader robots labeled 1 and 2 are required to achieve and keep a desired distance between each other. The rest of the robots are followers who do not know the reference velocity. Each follower follows its two neighbor robots and maintains a triangulation formation with them. Let  $\mathcal{N}_i$  denote the set of neighbors of robot  $i$ . In the paper, we assume that  $\mathcal{N}_1 = \{2\}$ ,  $\mathcal{N}_2 = \{1\}$ ,  $\mathcal{N}_3 = \{1, 2\}$ ,  $\mathcal{N}_4 = \{2, 3\}$ , and so on. The interaction directed graph for our setup is illustrated in Fig. 1 where an arc from node  $i$  to node  $j$  implies that robot  $i$  uses the relative position information of robot  $j$  for the purpose of formation control.

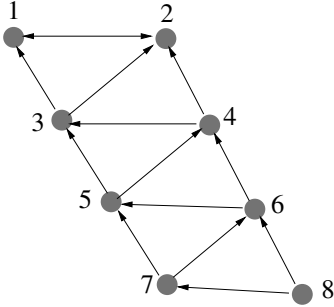


Fig. 1. Interaction directed graph.

In what follows, the notation  $\|\cdot\|$  is used to denote the Euclidean norm. Let  $d_{ij}$  be the desired distance between robot  $i$  and  $j$ . The problem of formation control for a group of robots with two leaders is described as follows.

**Problem:** Find a control law

$$u_i^L = f_i^L((x_j - x_i)_{j \in \mathcal{N}_i}, v_0)$$

for two leader robots 1 and 2 and a control law

$$u_i^F = f_i^F((x_j - x_i)_{j \in \mathcal{N}_i})$$

for each follow robot such that as  $t \rightarrow \infty$ ,

1)

$$\begin{aligned} \dot{x}_1(t) &\rightarrow v_0, \\ \dot{x}_2(t) &\rightarrow v_0, \\ \|x_1(t) - x_2(t)\| &\rightarrow d_{12}, \end{aligned}$$

2) for  $i \geq 3$ ,

$$\begin{aligned} \|x_i(t) - x_{i-2}(t)\| &\rightarrow d_{(i-2)i}, \\ \|x_i(t) - x_{i-1}(t)\| &\rightarrow d_{(i-1)i}. \end{aligned}$$

Next let's introduce the concept of formation. Consider a distance constraint function

$$f(x) = (\dots, \|x_i - x_k\| - d_{ik}, \|x_i - x_j\| - d_{ij}, \dots).$$

We say a group of  $n$  robots is in a formation specified by  $\dots, d_{ik}, d_{ij}, \dots$  if their states  $x$  satisfy  $f(x) = 0$ . Now let's denote the formation specified by  $d_{12}, d_{(i-1)i}, d_{(i-2)i}$  ( $i = 3, \dots, n$ ) in our problem by  $\mathbb{F}$ . To make the formation realizable, the desired distances must satisfy the following triangle inequality constraints.

$$\begin{aligned} d_{(i-2)(i-1)} &< d_{(i-2)i} + d_{(i-1)i}, \\ d_{(i-2)i} &< d_{(i-2)(i-1)} + d_{(i-1)i}, \\ d_{(i-1)i} &< d_{(i-2)(i-1)} + d_{(i-2)i}. \end{aligned}$$

Note that the formation  $\mathbb{F}$  in our problem is rigid, but not globally rigid. For example, the formation  $\mathbb{F}$  shown in Fig. 2(a) has a discontinuous deformation in Fig. 2(b) with the same distance constraints.

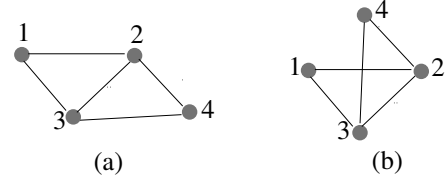


Fig. 2. Two rigid formations with the same distance constraints.

Finally, we introduce the stability notions of a formation.

**Definition 2.1:** A formation corresponding to  $f(x^*) = 0$  is called

- stable if for each  $\varepsilon > 0$ , there is  $\delta(\varepsilon) > 0$  such that  $\|f(x(0)) - f(x^*)\| < \delta$  implies  $\|f(x(t)) - f(x^*)\| < \varepsilon$  for all  $t > 0$ .
- unstable if it is not stable.
- asymptotically stable if it is stable and a constant  $c > 0$  can be chosen such that  $\|f(x(0)) - f(x^*)\| < c$  implies  $\lim_{t \rightarrow \infty} \|f(x(t)) - f(x^*)\| = 0$ .

## III. CONTROL SYNTHESIS

In this section, we synthesize control laws for the leaders as well as the followers to solve the formation control problem. Firstly, we present the control law for the two leader robots. Secondly, we propose an adaptive control law for the follower robots and investigate the convergence and stability properties.

### A. Control Law for Leader Robots

In this subsection, we study control laws for the two leader robots. Since the motion of these two leader robots is not affected by their followers, we can analyze their dynamical behavior independently. Recall that the reference velocity  $v_0$  is known only to these two leader robots. We then design the following control law using only locally sensed relative position information,

$$\begin{aligned} u_1^L &= v_0 + (x_2 - x_1)(\|x_2 - x_1\|^2 - d_{12}^2), \\ u_2^L &= v_0 + (x_1 - x_2)(\|x_1 - x_2\|^2 - d_{12}^2). \end{aligned} \quad (2)$$

With the above control, we then have the following dynamics for these two leader robots,

$$\begin{aligned}\dot{x}_1 &= v_0 + (x_2 - x_1)(\|x_2 - x_1\|^2 - d_{12}^2), \\ \dot{x}_2 &= v_0 + (x_1 - x_2)(\|x_1 - x_2\|^2 - d_{12}^2).\end{aligned}\quad (3)$$

Next we show that with the control law above, these two leader robots will eventually converge to a formation with the desired distance  $d_{12}$  and move with the reference velocity  $v_0$  if they are not initially coincident.

**Theorem 3.1:** For two leader robots with the control law (2), if  $x_1(0) \neq x_2(0)$ , then

$$\begin{aligned}\|x_1(t) - x_2(t)\| &\rightarrow d_{12} \\ \dot{x}_1(t) &\rightarrow v_0 \text{ exponentially as } t \rightarrow \infty. \\ \dot{x}_2(t) &\rightarrow v_0\end{aligned}$$

**Proof:** Let  $z_{12} = x_1 - x_2$  and  $e_{12} = \|z_{12}\|^2 - d_{12}^2$ . It holds that

$$\dot{z}_{12} = \dot{x}_1 - \dot{x}_2 = -2z_{12}e_{12}. \quad (4)$$

For the remaining, we refer to the proof of Lemma 4 in [10] from which one obtains  $e_{12} \rightarrow 0$  exponentially fast. Thus,  $\|z_{12}(t)\|$  tends to  $d_{12}$  exponentially and  $\dot{x}_1(t) \rightarrow v_0$ ,  $\dot{x}_2(t) \rightarrow v_0$  exponentially. ■

The bidirectional information flow between the two leader robots removes the rotational degree of freedom from the formation.

### B. Control Law for Follower Robots

In the last subsection we have discussed a control law for the two leader robots. In this subsection we investigate control laws for the follower robots. Firstly, we consider the control for robot 3 to see how it achieves a triangular formation with the two leader robots 1 and 2. This is critical since it will serve as the base to obtain control laws for all the other follower robots. We then generalize the control law and extend it to any follower robot  $i$  ( $i \geq 3$ ).

As the reference velocity is not known by the follower robots, we consider an adaptive control. To be more specific, for robot 3, we use the following control utilizing only the relative position information about robots 1 and 2:

$$\begin{aligned}\dot{\theta}_3 &= (x_1 - x_3)(\|x_1 - x_3\|^2 - d_{13}^2) \\ &\quad + (x_2 - x_3)(\|x_2 - x_3\|^2 - d_{23}^2), \\ u_3^F &= \theta_3 + (x_1 - x_3)(\|x_1 - x_3\|^2 - d_{13}^2) \\ &\quad + (x_2 - x_3)(\|x_2 - x_3\|^2 - d_{23}^2).\end{aligned}\quad (5)$$

Introduce the coordinate transformation

$$z_{13} = x_1 - x_3, \quad z_{23} = x_2 - x_3,$$

and let

$$e_{13} = \|z_{13}\|^2 - d_{13}^2, \quad e_{23} = \|z_{23}\|^2 - d_{23}^2.$$

Then we obtain

$$\begin{aligned}\dot{z}_{13} &= \dot{x}_1 - (\theta_3 + z_{13}e_{13} + z_{23}e_{23}), \\ \dot{z}_{23} &= \dot{x}_2 - (\theta_3 + z_{13}e_{13} + z_{23}e_{23}), \\ \dot{\theta}_3 &= z_{13}e_{13} + z_{23}e_{23},\end{aligned}\quad (6)$$

where  $\dot{x}_1 = v_0 - z_{12}e_{12}$  and  $\dot{x}_2 = v_0 + z_{12}e_{12}$  from (3).

We first show that  $z_{13}e_{13} + z_{23}e_{23}$  converges to 0 and  $\theta_3$  converges to  $v_0$  as  $t \rightarrow \infty$ . That is, the solution to the above system approaches the equilibrium set.

We recall the Barbalat's Lemma, which is helpful in the proof of the main result.

**Lemma 3.1:** (Barbalat's Lemma) [18] Let  $\varphi : \mathbb{R} \rightarrow \mathbb{R}$  be a uniformly continuous function on  $[0, \infty)$ . Suppose that  $\lim_{t \rightarrow \infty} \int_0^t \varphi(\tau) d\tau$  exists and is finite. Then,  $\varphi(t) \rightarrow 0$  as  $t \rightarrow \infty$ .

**Theorem 3.2:** For robot 3 with the control law (5),

$$\begin{aligned}z_{13}(t)e_{13}(t) + z_{23}(t)e_{23}(t) &\rightarrow 0 \\ \theta_3(t) &\rightarrow v_0 \text{ as } t \rightarrow \infty. \\ \dot{x}_3(t) &\rightarrow v_0\end{aligned}$$

A sketch of the proof is given below due to the space limitation. The readers refer to [17] for details.

**Proof of Theorem 3.2:** Define a continuously differentiable function

$$V = \frac{1}{4}e_{13}^2 + \frac{1}{4}e_{23}^2 + \frac{1}{2}\|\theta_3\|^2.$$

Taking the time derivative of  $V$  along the solution of system (6), one obtains

$$\begin{aligned}\dot{V} &= \frac{1}{2}e_{13}\dot{e}_{13} + \frac{1}{2}e_{23}\dot{e}_{23} + \theta_3^T \dot{\theta}_3 \\ &= \begin{bmatrix} e_{13}z_{13}^T & e_{23}z_{23}^T \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} \\ &\quad - \|e_{13}z_{13} + e_{23}z_{23}\|^2 \\ &\leq \left\| \begin{bmatrix} e_{13}z_{13}^T & e_{23}z_{23}^T \end{bmatrix} \right\| \left\| \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} \right\| \\ &\quad - \|e_{13}z_{13} + e_{23}z_{23}\|^2.\end{aligned}$$

From the expression of  $V$ , we know that  $e_{13}(t)$ ,  $e_{23}(t)$ ,  $\theta_3(t)$  are bounded and so are  $z_{13}(t)$  and  $z_{23}(t)$ . Since  $\theta_3 = z_{13}e_{13} + z_{23}e_{23}$ , one gets

$$\theta_3(t) = \int_0^t [z_{13}(\tau)e_{13}(\tau) + z_{23}(\tau)e_{23}(\tau)] d\tau + \theta_3(0).$$

Note that  $\theta_3(t)$  is bounded, so  $\lim_{t \rightarrow \infty} \int_0^t (z_{13}(\tau)e_{13}(\tau) + z_{23}(\tau)e_{23}(\tau)) d\tau$  exists and is finite. Moreover,  $z_{13}e_{13} + z_{23}e_{23}$  is uniformly continuous. Hence, from Lemma 3.1,

$$z_{13}(t)e_{13}(t) + z_{23}(t)e_{23}(t) \rightarrow 0 \text{ as } t \rightarrow \infty.$$

Next we prove that  $\theta_3(t) \rightarrow v_0$ . Since  $\theta_3(t)$  is bounded and  $\dot{\theta}_3(t) \rightarrow 0$ , we obtain  $\theta_3(t) \rightarrow a$  where  $a$  is a constant. From Theorem 3.1, we know  $e_{12}(t) \rightarrow 0$  as  $t \rightarrow \infty$ . Together with  $z_{13}(t)e_{13}(t) + z_{23}(t)e_{23}(t) \rightarrow 0$ , it follows from (6) that  $\dot{z}_{13}(t) \rightarrow (v_0 - a)$  and  $\dot{z}_{23}(t) \rightarrow (v_0 - a)$ . If  $a \neq v_0$ ,  $z_{13}(t)$  and  $z_{23}(t)$  would tend to  $\infty$ , which contradicts with the conclusion that  $z_{13}(t)$  and  $z_{23}(t)$  are bounded. Therefore, the constant  $a$  must be  $v_0$ . That is,  $\theta_3(t) \rightarrow v_0$ .

Finally, from (5) we obtain  $\dot{x}_3(t) \rightarrow v_0$ . ■

Note that  $z_{13}e_{13} + z_{23}e_{23} = 0$  means either

- (1)  $e_{13} = 0, e_{23} = 0$ , or
- (2)  $z_{13}e_{13} + z_{23}e_{23} = 0$  but  $e_{13}$  or  $e_{23} \neq 0$ .

For the first case, if  $e_{12} = 0$ , it corresponds to a desired triangular formation. That is, the three robots 1, 2 and 3

form a triangle with desired edge lengths  $d_{12}$ ,  $d_{13}$  and  $d_{23}$ . The second case corresponds to a *collinear formation*. That is, the three robots are positioned on a line (see Fig. 3).

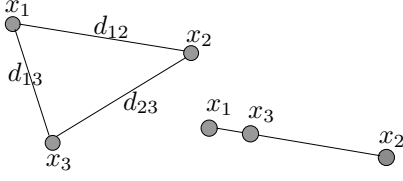


Fig. 3. Triangular formation (left) and collinear formation (right).

Recall from Theorem 3.1 that for two leader robots with the control law (2), if  $x_1(0) \neq x_2(0)$ , then  $e_{12}(t) \rightarrow 0$ . And Theorem 3.2 tells us that robot 3 converges to either a triangular formation or a collinear formation with robots 1 and 2. Next we show that the triangular formation is asymptotically stable while the collinear formation is unstable.

**Theorem 3.3:** For robots 1, 2, and 3 with the control laws (2) and (5), the triangular formation is asymptotically stable.

The proof of Theorem 3.3 requires the following two lemmas.

**Lemma 3.2:** [6] Let  $\mathcal{E}$  be a  $k$ -dimensional equilibrium manifold of  $\dot{x} = f(x)$ . Let  $J_f(x_0)$  be the Jacobian matrix at  $x_0$ . If, for all  $x_0 \in \mathcal{E}$ ,  $J_f(x_0)$  has all stable eigenvalues, except for  $k$  eigenvalues at zero, then  $\mathcal{E}$  is locally asymptotically stable.

**Lemma 3.3:** [17] For any two vectors  $z_1, z_2 \in \mathbb{R}^2$ , the following holds:

$$\|z_1\|^2 + \|z_2\|^2 \geq 2 \det[z_1 \ z_2].$$

**Proof of Theorem 3.3:** We prove it using Lyapunov's indirect method. For these three robots, the overall system is given in (4) and (6). Then the Jacobian matrix at any state satisfying  $e_{12} = 0$  is calculated and has the following block lower triangular matrix form

$$J = \begin{pmatrix} -2A & 0 \\ E & D \end{pmatrix}, \quad (7)$$

where

$$A = 2z_{12}z_{12}^T, \quad E = (-A \ A \ 0)^T, \\ D = \begin{pmatrix} -e_{13}I_2 - B & -e_{23}I_2 - C & -I_2 \\ -e_{13}I_2 - B & -e_{23}I_2 - C & -I_2 \\ e_{13}I_2 + B & e_{13}I_2 + C & 0 \end{pmatrix},$$

and

$$B = 2z_{13}z_{13}^T, \quad C = 2z_{23}z_{23}^T.$$

For the triangular formation (namely,  $e_{12} = e_{13} = e_{23} = 0$ ), the matrix  $D$  then becomes

$$D = \begin{pmatrix} -B & -C & -I_2 \\ -B & -C & -I_2 \\ B & C & 0 \end{pmatrix}.$$

One can check that from Routh's criterion and Lemma 3.3, it is obtained that the characteristic equation for the matrix  $D$

have two zero eigenvalues and all the other four eigenvalues have negative real part.

On the other hand, note that the 2-by-2 matrix  $A$  is of rank one and is positive semi-definite, so  $-2A$  has one zero eigenvalue and the other eigenvalue has negative real part. Hence, the Jacobian matrix  $J$  at any state satisfying  $e_{12} = e_{13} = e_{23} = 0$  has three zero eigenvalues and five eigenvalues with negative real part. Moreover, it can be checked that the equilibrium manifold corresponding to the triangular formation (namely,  $\{(z_{12}, z_{13}, z_{23}, \theta_3) | e_{12} = e_{13} = e_{23} = 0, \theta_3 = v_0\}$ ) is of dimension three. Then, by Lemma 3.2, it follows that the triangular formation is asymptotically stable. ■

Next, we show that the collinear formation is unstable. Before presenting the result, we introduce a lemma first, which will be used in the proof.

**Lemma 3.4:** [10] If three robots are in the collinear formation, then  $e_{13} + e_{23} < 0$ .

**Theorem 3.4:** For robots 1, 2, and 3 with the control laws (2) and (5), the collinear formation is unstable.

**Proof:** For collinear formation,  $z_{13}e_{13} + z_{23}e_{23} = 0$  but  $e_{13}$  or  $e_{23} \neq 0$ . Without loss generality, say  $e_{13} \neq 0$ . Then we write

$$z_{13} = -\frac{e_{23}}{e_{13}}z_{23}$$

and substitute it into (7). Thus, the Jacobian matrix at any state corresponding to collinear formation is still of the form (7) but the sub-block  $D$  in the Jacobian matrix  $J$  becomes

$$D = \begin{pmatrix} -e_{13}I_2 - (\frac{e_{23}}{e_{13}})^2C & -e_{23}I_2 - C & -I_2 \\ -e_{13}I_2 - (\frac{e_{23}}{e_{13}})^2C & -e_{23}I_2 - C & -I_2 \\ e_{13}I_2 + (\frac{e_{23}}{e_{13}})^2C & e_{23}I_2 + C & 0 \end{pmatrix},$$

where  $C$  is still the same. It can be checked that one of two eigenvalues must have positive real part [17]. Hence, the collinear formation is unstable. ■

We have now investigated the convergence and stability properties of two possible formations (namely, the triangular formation and the collinear formation) for three mobile robots. In what follows, we generalize the adaptive control law to any follower robot. That is, for any robot  $i$  ( $i \geq 3$ ), let the control law be

$$\begin{aligned} \dot{\theta}_i &= (x_{i-2} - x_i)(\|x_{i-2} - x_i\|^2 - d_{(i-2)i}^2) \\ &\quad + (x_{i-1} - x_i)(\|x_{i-1} - x_i\|^2 - d_{(i-1)i}^2), \\ u_i^F &= \theta_i + (x_{i-2} - x_i)(\|x_{i-2} - x_i\|^2 - d_{(i-2)i}^2) \\ &\quad + (x_{i-1} - x_i)(\|x_{i-1} - x_i\|^2 - d_{(i-1)i}^2), \end{aligned} \quad (8)$$

which uses only the relative position information of its precedent two neighbor robots according to their labels.

Similarly, define the relative positions

$$z_{(i-2)i} = x_{i-2} - x_i, \quad z_{(i-1)i} = x_{i-1} - x_i,$$

and let

$$\begin{aligned} e_{(i-2)i} &= \|z_{(i-2)i}\|^2 - d_{(i-2)i}^2, \\ e_{(i-1)i} &= \|z_{(i-1)i}\|^2 - d_{(i-1)i}^2. \end{aligned}$$

We then obtain the dynamics

$$\begin{aligned}\dot{z}_{(i-2)i} &= \dot{x}_{i-2} - (\theta_i + z_{(i-2)i}e_{(i-2)i} + z_{(i-1)i}e_{(i-1)i}), \\ \dot{z}_{(i-1)i} &= \dot{x}_{i-1} - (\theta_i + z_{(i-2)i}e_{(i-2)i} + z_{(i-1)i}e_{(i-1)i}), \\ \dot{\theta}_i &= z_{(i-2)i}e_{(i-2)i} + z_{(i-1)i}e_{(i-1)i},\end{aligned}\quad (9)$$

where  $\dot{x}_{i-2}$  and  $\dot{x}_{i-1}$  are the position dynamics of its precedent two neighbor robots. Note that the dynamics above have exactly the same form as (6). So by a similar argument, one is able to obtain the following result for any follower robot  $i$  ( $i \geq 3$ ). That is, if the precedent two robots are not coincident, then robot  $i$  converges to form a triangular formation or a collinear formation with the precedent two neighbor robots.

*Corollary 3.1:* Under the control law (8),

$$\begin{aligned}z_{(i-2)i}(t)e_{(i-2)i}(t) + z_{(i-1)i}(t)e_{(i-1)i}(t) &\rightarrow 0 \\ \theta_i(t) &\rightarrow v_0 \quad \text{as } t \rightarrow \infty. \\ \dot{x}_i(t) &\rightarrow v_0\end{aligned}$$

The Jacobian matrix of the overall system evaluated at any state satisfying  $e_{12} = 0$  has the following form

$$J = \left( \begin{array}{c|cccc} -2A & 0 & & & \\ \hline & D_3 & 0 & \cdots & 0 \\ * & * & D_4 & \ddots & \vdots \\ & \vdots & \ddots & \ddots & 0 \\ & 0 & \cdots & * & D_n \end{array} \right), \quad (10)$$

where  $A = 2z_{12}z_{12}^T$  and for  $i \geq 3$ ,

$$D_i = \begin{pmatrix} -e_{(i-2)i}I_2 - B_i & -e_{(i-1)i}I_2 - C_i & -I_2 \\ -e_{(i-2)i}I_2 - B_i & -e_{(i-1)i}I_2 - C_i & -I_2 \\ e_{(i-2)i}I_2 + B_i & e_{(i-1)i}I_2 + C_i & 0 \end{pmatrix},$$

$$B_i = z_{(i-2)i}z_{(i-2)i}^T, \quad C_i = z_{(i-1)i}z_{(i-1)i}^T.$$

Note that the Jacobian matrix  $J$  is of the block lower triangular form. So we are able to check the stability from the location of  $D_i$ 's eigenvalues for  $i = 3, \dots, n$ . Hence, using the same technique as for Theorem 3.3 and Theorem 3.4, the following result is obtained.

*Corollary 3.2:* For a group of  $n$  robots with the control laws (2) and (8), the formation  $\mathbb{F}$  specified in our problem is asymptotically stable, and any formation with any three robots being collinear is unstable.

As the interaction directed graph does not introduce any cycle when adding more and more follower robots, the approach is applicable to formations with a large number of robots. However, the transient response of each follower robot may differ depending on how far away it is from the leaders in terms of the path length in the interaction graph. It is of interest to study how the transient response is amplified from the first follower to the  $n$ th follower as  $n$  tends to  $\infty$ . Ideas like the mesh stability [19] might be useful.

#### IV. SIMULATION

In this section, we present several simulations to illustrate our results.

First, we simulate three robots using the control laws (2) and (5) for two leader robots and one follower robot. The

reference velocity  $v_0$  is piecewise constant and it changes its value at some time. The initial positions of three robots are randomly generated. The simulated trajectories of three robots under the control laws are given in Fig. 4. They form a triangular formation and move in the plane. After the abrupt change of the reference velocity, three robots can be recovered to the triangular formation and move as a whole again with the new velocity. Second, we present a simulation in Fig. 5 for twenty robots that achieve a formation in motion.

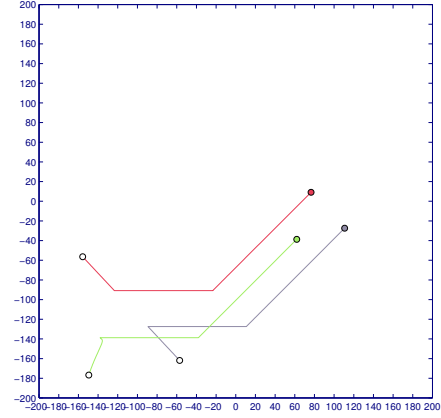


Fig. 4. Three robots form a triangular formation and move in the plane.

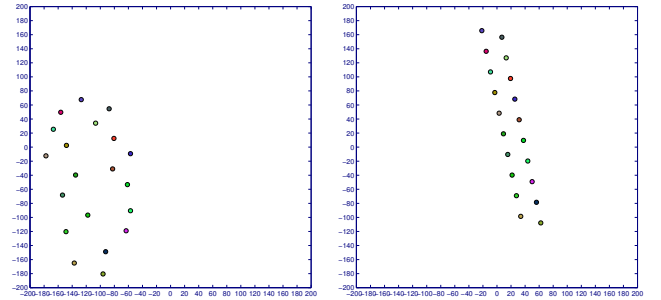


Fig. 5. Initial distribution (left) and final formation (right) of 20 robots.

Notice that for three robots there are two distinct triangular formations satisfying the prescribed distance constraints (see Fig. 2). One triangular formation is shown in Fig. 3 (left), which is referred to as a *positively-oriented triangular formation*, and the other that is flipped over by Fig. 3 (left) is referred to as a *negatively-oriented triangular formation*. Currently, we are not sure yet for what initial configuration three robots will form a positively-oriented triangular formation rather than the other one and vice versa. Hence, if the number of robots is greater than three and all the desired distances are equal, robot  $i$  and robot  $i - 3$  might overlap under this control strategy. For instance, for some initial distribution, the simulated trajectories of four robots are shown in Fig. 6, where they form a quadrangular formation and move in the plane, but for another initial distribution,

the trajectories are shown Fig. 7 where two of them overlap. However, both achieve a rigid formation.

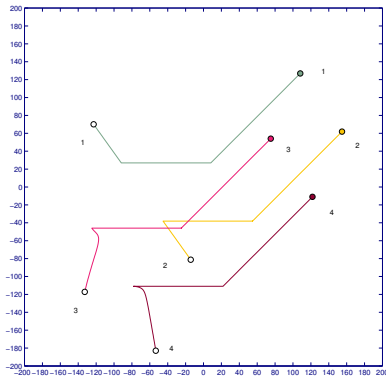


Fig. 6. Four robots form a quadrangular formation and move in the plane.

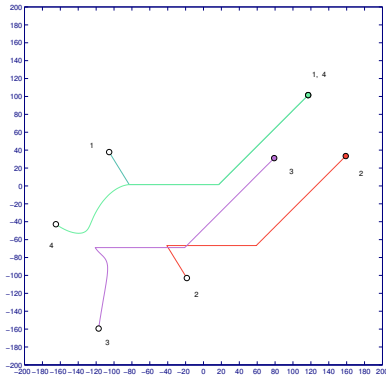


Fig. 7. Four robots form a formation but two of them overlap.

## V. CONCLUSION AND FUTURE WORK

In this paper, we address the problem of adaptive formation control. The mobile robots in the group are controlled to move as a rigid formation with a prescribed velocity. There are two leaders in the robot group, who know the reference velocity information, while all the other robots do not. Each leader robot has the other leader robot as its neighbor and controls the distance separation between them. Every follower robot has exactly two neighbors according to their labels and it determines its movement strategy using only local knowledge of the relative positions of its neighbors. A control law is designed for the leader robots first. Then an adaptive control law is investigated for every follower robot to achieve a triangular formation with its precedent two neighbor robots. Using Barbalat's lemma, the convergence properties are established. The stability properties are analyzed by the Lyapunov indirect method.

The control strategy developed in the paper can be applied to a large number of robots moving in a formation with a simple cascade neighbor relationship. However, in this setup,

one open issue is that the tracking (spacing) errors might be amplified downstream from robot to robot in the presence of disturbance at the leader robots, which is related to the mesh stability problem.

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